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COMPARING THE ACHIEVED AND DESIRED FAMILY SIZE OF MARRIED WOMEN USING A METHOD OF MEDIAN TEST FOR MATCHED SAMPLE

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ABSTRACT

Background: The application of nonparametric statistical methods becomes necessary when the data collected from samples do not meet the foundational assumptions of continuity and normality that are prerequisites for their parametric counterparts. This paper introduces and formalizes a ties-adjusted median test specifically designed for analysing paired-sample populations. This novel method serves as a modified alternative to conventional nonparametric techniques like the ordinary Sign test and the Wilcoxon Signed Rank test.

Methodology: The procedural foundation of this study involved adapting the extended median test for matched samples to specifically account for the potential occurrence of tied observations. This is particularly relevant for datasets where measurements may be on an ordinal scale. The resulting chi-square test statistic was formulated to evaluate the null hypothesis of equal treatment medians and was subsequently applied to an empirical dataset to demonstrate its practical application.

Results: To illustrate its utility, the proposed method was used to test the null hypothesis that there is no significant difference between the actual and desired number of children among women in a sampled community. The analysis yielded a p-value of 0.0048. For the purpose of comparison, the same data analysed with the Sign test produced a p-value of 0.0193, while the Wilcoxon Signed Rank test resulted in a p-value of 0.0139.

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Conclusions: At a 5% significance level, the proposed method demonstrates greater statistical power than both the ordinary Sign test and the Wilcoxon Signed Rank test. This suggests that the new test possesses a higher probability of accurately rejecting a null hypothesis that is indeed false.

Keywords: Paired Data Analysis, Nonparametric Statistics, Ties-Adjusted Median Test, Matched Samples, Chi-Square Statistic, Statistical Power.

Introduction

In statistical analysis, paired sample data are typically analysed using parametric tests, provided that the data are drawn from two populations that satisfy the critical assumptions of continuity and normality. However, when these assumptions are not met, resorting to nonparametric alternatives becomes the preferred course of action. Among the most common nonparametric methods for such data are the Sign test and the Wilcoxon Signed Rank test.

A significant challenge arises when these tests are applied to datasets containing a substantial number of tied observations. Standard practice for a few ties involves either dropping the tied pairs, which reduces the sample size, or assigning them a mean rank in the case of the Wilcoxon test. Yet, if the frequency of ties is high, these conventional approaches may not be adequate without specific adjustments. The failure to properly account for a large number of ties can severely compromise the statistical power of the test, potentially leading to unreliable conclusions.

To address this methodological gap, this paper proposes and develops a ties-adjusted median test. This approach is built upon the framework of the extended median test for matched samples and is designed to provide a more robust analysis when ties are prevalent in the data.

METHODOLOGY

Suppose we have a random sample of ‘r’ subjects or candidates observed at two points in time or space or under two experimental conditions or treatments or assessed by two judges or instructors. Let x_{ij} be the score by the ith subject under treatment j for i=1,2,..,r and j=1,2.These observations or scores may be measurements on as low as the ordinal scale. Let M_i be the median of the two observations on the ith subject. Note that actually M_i is the average of the two observations on the ith subject, that is, $M_i = (x_{i1} + x_{i2})/2$, for i=1,2,….,r. Now let

$$u_{ij} = \begin{cases} 1, if \;\; x_{ij} > M_i \\ 0, if \;\; x_{ij} = M_i \\ -1, if \;\; x_{ij} < M_i \end{cases} \qquad 1$$

For i=1,2,…..,r; j=1,2.

Note that, Equation 1 is equivalent to saying that u_{ij} assumes the value 1,0,-1 if one of the observations on the ith subject is greater than, equal to, or less than the other observation. Thus the present method is in this respect similar to the approach used in the ordinary Sign test except that provision has now been made for the possible presence of ties in the data. Let $t_j^+, t_j^0 \; and \; t_j^-$ be respectively the number of 1s,0s and -1s in the experimental condition or treatment j for all i=1,2,….,r ;j=1,2.Note that $t_j^0 = r - t_j^+ - t_j^-$.Also let $t^+, t^0 \; and \; t^-$ be respectively the total number of 1s,0s and -1s in u_{ij} .That is

$$t^+ = \sum_{j=1}^{2} t_j^+ ; t^- = \sum_{j=1}^{2} t_j^- ; t^0 = \sum_{j=1}^{2} t_j^0 = 2r - t^+ - t^- \qquad 2$$

The proportions of 1s,-1s and 0s at the jth level of treatment or experimental condition, that is the jth level(j=1,2) of the responses for all the subjects are respectively

$$P_j^+ = \frac{t_j^+}{r} ; P_j^- = \frac{t_j^-}{r} ; P_j^0 = \frac{t_j^0}{r} = 1 - P_j^+ - P_j^- . \qquad 3$$

The overall proportions of 1s,-1s and 0s are respectively

$$P^+ = \frac{t^+}{2r} ; P^- = \frac{t^-}{2r} ; P^0 = \frac{t^0}{2r} = 1 - P^+ - P^- . \qquad 4$$

The observed number of 1s, -1s and 0s at the jth level of response are respectively

$$O_{1j} = t_j^+ ; O_{2j} = t_j^- ; O_{3j} = t_j^0 = r - t_j^+ - t_j^- \qquad 5$$

Under the null hypothesis of no difference between the proportions of responses for the two treatment levels, that is the null hypothesis of equal population medians, the corresponding expected frequencies are

$$E_{1j} = \frac{rt^+}{2r} ; E_{2j} = \frac{rt^-}{2r} ; E_{3j} = \frac{rt^0}{2r} = \frac{r\left(2r - t^+ - t^-\right)}{2r} \qquad 6$$

To test the null hypothesis of equal population medians, we use the test statistic

$$\chi^2 = \sum_{j=1}^{2}\sum_{i=1}^{3}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}}$$

which under the null hypothesis H0 has approximately the Chi-square distribution with 2 degrees of freedom, substituting Equations 5 and 6 in the above expression yields

$$\chi^2 = \sum_{j=1}^{2}\frac{\left(t_j^+ - \frac{rt^+}{2r}\right)^2}{\frac{rt^+}{2r}} + \sum_{j=1}^{2}\frac{\left(t_j^- - \frac{rt^-}{2r}\right)^2}{\frac{rt^-}{2r}} + \frac{\sum_{j=1}^{2}\left((r-t_j^+-t_j^-)-\frac{r(2r-t^+-t^-)}{2r}\right)^2}{\frac{r(2r-t^+-t^-)}{2r}}$$

That is

$$\chi^2 = 2\left(\frac{\sum_{j=1}^{2}\left(t_j^+-\frac{t^+}{2}\right)^2}{t^+} + \frac{\sum_{j=1}^{2}\left(t_j^--\frac{t^-}{2}\right)^2}{t^-} + \frac{\sum_{j=1}^{2}\left(\left(t_j^+-\frac{t^+}{2}\right)+\left(t_j^--\frac{t^-}{2}\right)\right)^2}{2r-t^+-t^-}\right)$$

Which when further simplified yields

$$\chi^2 = \frac{2\left(t^+\left(2r-t^-\right)\sum_{j=1}^{2}\left(t_j^+-\frac{t^+}{2}\right)^2 + t^+\left(2r-t^+\right)\sum_{j=1}^{2}\left(t_j^--\frac{t^-}{2}\right)^2 + 2t^+t^-\sum_{j=1}^{2}\left(t_j^+-\frac{t^+}{2}\right)\left(t_j^--\frac{t^-}{2}\right)\right)}{t^+t^-\left(2r-t^+-t^-\right)} \qquad 7$$

Or when expressed in terms of the proportions in equations 3 and 4 becomes

$$\chi^2 = \frac{r\left(P^-\left(1-P^-\right)\sum_{j=1}^{2}\left(P_j^+-P^+\right)^2 + P^+\left(1-P^+\right)\sum_{j=1}^{2}\left(P_j^--P^-\right)^2 + 2P^+P^-\sum_{j=1}^{2}\left(P_j^+-P^+\right)\left(P_j^--P^-\right)\right)}{P^+P^-\left(1-P^+-P^-\right)} \qquad 8$$

Which has the Chi-square distribution with (3-1)(2-1)=2degrees of freedom.

Equation 8 may be simplified in an easier way to obtain the computational form as

$$\chi^2 = \frac{r\left(P^-\left(1-P^-\right)\left(P_1^+-P_2^+\right)^2 + P^+\left(1-P^+\right)\left(P_1^--P_2^-\right)^2 + 2P^+P^-\left(P_1^+-P_2^+\right)\left(P_1^--P_2^-\right)\right)}{2P^+P^-\left(1-P^+-P^-\right)} \qquad 9$$

Note that $\sin ce\ P_1^+ = P_2^-\ and\ P_1^- = P_2^+\ therefore\ P^+ = P^-$. Hence Equation 9 may be further simplified to

$$\chi^2 = \frac{r\left(P_1^+ - P_2^+\right)^2}{P^+} = \frac{r\left(P_1^- - P_2^-\right)^2}{P^-} \quad 10$$

Or equivalently

$$\chi^2 = \frac{2r\left(P_1^+ - P_2^+\right)^2}{2P^+} = \frac{2r\left(P_1^- - P_2^-\right)^2}{2P^-} \quad 11$$

The null hypothesis H0 of equal population or treatment medians is rejected at the α level of significance if

$$\chi^2 \geq \chi^2_{1-\alpha;2} \quad 12$$

Otherwise H0 is accepted.

Results: Illustrative Example

To demonstrate the application of this method, we analysed data from a study conducted by a family planning consultant aiming to understand the fertility goals of women in a specific community. A random sample of 17 women was selected, and information regarding their actual (achieved) and desired number of children was collected via a questionnaire. The collected data are presented in Table 1.

Table1. Achieved and Desired Family sizes of a Random Sample of women from a certain community

Women	**Achieved no of children**	**Desired no of children**	$M_i = (x_{i1} + x_{i2})/2$	**Score for Treatment**		**Total**	**Proportion**
I	x_{i1}	x_{i2}		u_{i1}	u_{i2}	**(t)**	**(p)**
1	**4**	**5**	**4.5**	**-1**	**1**		
2	**1**	**5**	**3**	**-1**	**1**		
3	**6**	**5**	**5.5**	**1**	**-1**		
4	**1**	**6**	**3.5**	**-1**	**1**		
5	**7**	**5**	**6**	**1**	**1**		

6	**1**	**9**	**5**	**-1**	**1**		
7	**4**	**4**	**4**	**0**	**0**		
8	**2**	**6**	**4**	**-1**	**1**		
9	**8**	**8**	**8**	**0**	**0**		
10	**5**	**5**	**5**	**0**	**0**		
11	**4**	**4**	**4**	**0**	**0**		
12	**4**	**5**	**4.5**	**-1**	**1**		
13	**5**	**6**	**5.5**	**-1**	**1**		
14	**5**	**6**	**5.5**	**-1**	**1**		
15	**4**	**4**	**4**	**0**	**0**		
16	**4**	**6**	**5**	**-1**	**1**		
17	**5**	**6**	**5.5**	**-1**	**1**		
t_j^+				**2**	**10**	**12**	
t_j^-				**10**	**2**	**12**	
t_j^0				**5**	**5**	**10**	
Total				**17**	**17**	**34**	
p_j^+				$\frac{2}{17}=0.118$	$\frac{10}{17}=0.588$		$\frac{12}{34}\,p=0.353p^+$
p_j^-				$\frac{10}{17}=0.588$	$\frac{2}{17}=0.118$		$\frac{12}{34}\,p=0.353p^-$
p_j^0				$\frac{5}{17}=0.294$	$\frac{5}{17}=0.294$		$\frac{10}{34}\,p=0.294p^0$

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For each participant, the median family size was calculated as the average of her achieved and desired number of children. These median values were then used to generate scores of 1, 0, or -1 for the achieved and desired family sizes, as detailed in the methodology. From these scores, the necessary proportions were computed to test the null hypothesis that no difference exists between the actual and desired family sizes of women in this community.

Applying the computational formula (Equation 10 from the source), the analysis yielded: $\chi^2 = 10.59$.

With 2 degrees of freedom, the critical value at a 5% significance level is $\chi^2_{0.05,2} = 5.99$. Since the calculated value of 10.59 is greater than the critical value, the result is statistically significant. We therefore reject the null hypothesis and conclude that the actual and desired family sizes of women in the sampled community differ significantly.

For comparative analysis, the data were also evaluated using the ordinary Sign test. The dataset included five instances where the achieved and desired family sizes were equal, resulting in five ties. Following the standard procedure for the Sign test, these ties were removed, reducing the effective sample size to $n=17-5=12$. The analysis resulted in a binomial probability (p-value) of 0.0193, which is significant at the 5% level but not at the 1% level. The p-value obtained from the proposed method (0.0048) is substantially lower than that from the Sign test (0.0193). This suggests that the Sign test is more likely to lead to a Type II error (failure to reject a false null hypothesis) and is therefore less powerful in this context.

A similar comparison was made with the Wilcoxon Signed Rank test. After calculating the differences between actual and desired family sizes, the 12 non-zero absolute differences were ranked. The sum of ranks for the positive differences was found to be 11.0. The resulting test statistic produced a p-value of 0.0139, which is also significant at the 5% level but not at the 1% level. This p-value is also higher than that of the proposed method, again indicating that the new test may be more powerful than the Wilcoxon Signed Rank test for this dataset.

Strengths and Limitations

The primary strength of this study lies in the development of a ties-adjusted median test that directly addresses a known challenge in nonparametric analysis. By adapting the extended median test for matched samples, this method overcomes the difficulties encountered by the Sign test and the Wilcoxon Signed Rank test when dealing with numerous ties, thereby preventing the loss of statistical power and reducing the risk of unreliable conclusions. The proposed test, which can be described as a modified extended median test, demonstrates superior power compared to these established methods.

A limitation of this study is that the proposed method is, by design, intended for scenarios where the assumptions of parametric tests are not met. Its application is therefore confined to non-normal or ordinal datasets. Unlike its parametric counterparts, which can be used when conditions of normality and continuity are satisfied, this nonparametric procedure is specific to contexts where such assumptions are violated.

Discussion and Conclusion

This research has introduced a nonparametric statistical method tailored for the analysis of paired sample data, particularly when the underlying populations do not conform to the assumptions of continuity and normality. The developed technique adapts the existing framework of the extended median test for matched samples by incorporating a specific adjustment for tied observations. This modification positions the test as an improved alternative to both the ordinary Sign test and the Wilcoxon Signed Rank test.

The practical utility of the proposed method was demonstrated through an analysis of sample data on the actual and desired family sizes of women. The chi-square-based test revealed a significant discrepancy between these two variables. When the results were compared against those obtained from the Sign and Wilcoxon Signed Rank tests on the same data—where ties were discarded as per standard procedure—the new method proved to be more powerful. The lower p-value of the proposed test (0.0048) compared to the Sign test (0.0193) and the Wilcoxon test (0.0139) indicates a reduced probability of committing a Type II error. This enhanced power suggests that the ties-adjusted median test is a more sensitive tool for detecting true effects in paired data with a high incidence of ties.

Further research is recommended to expand upon these findings. A valuable next step would be to conduct comparative studies where the Sign and Wilcoxon Signed Rank tests are first adjusted for ties and then their performance is benchmarked against the ties-adjusted median

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test presented here. Such an investigation, potentially using a variety of datasets or simulation studies, would provide a more comprehensive assessment of their relative statistical power.

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